

# Concrete-Representational-Abstract Instructional Approach

## What Is the Concrete-Representational-Abstract (CRA) Instructional Approach?

CRA is an intervention for mathematics instruction that research suggests can enhance the mathematics performance of ALL students. It is a three-part instructional strategy, with each part building on the previous instruction.

The CRA instructional sequence consists of three stages: concrete, representation, and abstract:

- *Concrete*. In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (e.g., red and yellow chips, cubes, base-ten blocks, pattern blocks, fraction bars, and geometric figures).
- *Representational*. In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting.
- *Abstract*. At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, −, ×, ÷) to indicate addition, multiplication, or division.

### **Concrete**

The “doing” stage using concrete objects to model problems

### **Representational**

The “seeing” stage using representations of the objects to model problems

### **Abstract**

The “symbolic” stage using abstract symbols to model problems

## How does the CRA instructional strategy positively impact students?

1. CRA supports understanding underlying mathematical concepts **before** learning “rules.” It teaches conceptual understanding by connecting concrete understanding to abstract math processes.
2. By linking learning experiences from concrete-to-representational-to-abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.
3. CRA blends conceptual/procedural understanding in a structured way so that students learn both the “How” and the “Why” to the problem solving procedures they learn to do; and, they learn the “What,” that is they develop conceptual understanding of the mathematics concept that underlies the problem solving process.

# CONCRETE

Research-based studies show that students who use concrete materials:

- develop more precise and more comprehensive mental representations
- often show more motivation and on-task behavior
- understand mathematical ideas
- better apply these ideas to life situations

(Harrison & Harrison, 1986; Suydam & Higgins, 1977)

Some mathematical concepts for which structured concrete materials work well as a foundation to develop understanding of concepts are:

- early number relations
- place value
- computation
- fractions
- decimals
- measurement
- geometry
- money
- percentage
- number bases
- word problems
- probability and statistics

Examples of Manipulatives:

- Colored counters
- Beans
- Unifix cubes
- Golf tees
- Skittles or other different colored candy pieces
- Packaging popcorn
- Popsicle sticks/tongue depressors
- Base 10 cubes/blocks
- Fraction pieces
- Fraction strips
- Fraction blocks or stacks
- Geoboards

Suggestions for using manipulatives:

1. Talk with your students about how manipulatives help to learn math.
2. Set ground rules for using manipulatives; review those rules every time you use manipulatives.
3. Develop a system for storing manipulatives.
4. Allow time for your students to explore manipulatives before beginning instruction.
5. Encourage students to learn names of the manipulatives they use.
6. Provide students time to describe the manipulatives they use orally or in writing. Model this as appropriate.

# REPRESENTATIONAL

At the representational level of understanding, students learn to problem-solve by **drawing pictures**. The pictures students draw represent the concrete objects students manipulated when problem-solving at the concrete level.

It is appropriate for students to begin drawing solutions to problems as soon as they demonstrate they have mastered a particular math concept/skill at the concrete level. Students typically need practice solving problems through drawing.

When students learn to draw solutions, students:

- are provided an intermediate step where they begin transferring their concrete understanding toward an abstract level of understanding
- they gain the ability to solve problems independently
- begin to 'internalize' the particular problem-solving process
- reinforce the concrete understanding of the concept/skill because of the similarity of their drawings to the manipulatives they used previously at the concrete level

**Drawing is not a 'crutch' for students that they will use forever. It simply provides students an effective way to practice problem solving independently until they develop fluency at the abstract level.**

The use of tallies, dots, and circles are examples of simple drawings students can make. By replicating the movements students previously used with concrete materials, drawing simple representations of those objects supports students' evolving abstract understanding of the concept/skill. They replicate similar movements using slightly more abstract representations of the mathematics concept/skill.

# ABSTRACT

At this stage, the teacher models the mathematics concept at a **symbolic level**, using **only numbers, notation, and mathematical symbols** to represent the number of circles or groups of circles. The teacher uses operation symbols to indicate addition, multiplication, or division.

A student who problem-solves at the abstract level, does so *without* the use of concrete objects or without drawing pictures. Understanding math concepts and performing math skills at the abstract level requires students to do this with numbers and math symbols only. Abstract understanding is often referred to as, "doing math in your head". Completing math problems where math problems are written and students solve these problems using paper and pencil is a common example of abstract level problem solving.

If students are struggling with the abstract after successful mastery of concrete and representational then:

1. Re-reach the concept/skill at the concrete level using appropriate concrete objects.
2. Re-teach the concept/skill at representational level and provide opportunities for students to practice concept/skill by drawing solutions.
3. Provide opportunities for students to use language to explain their solutions and how they got them.

**"Student learning and mastery greatly depends on the number of opportunities a student has to respond. The more opportunities for successful practice that you provide, the more likely it is that**